___1___ 1.) How many positive integers are equal to the number of letters in the English spelling of the number? (For example, the word "fifteen" has only 7 letters, so it would not be counted as one of the words we are looking for). Do not count spaces or special characters… only letters!

4,160,000 2.) Initially, a certain state was considering license plates with one capital letter followed by five single-digit integers (0 thru 9). However, they ultimately decided to go with two capital letters followed by four single-digit integers. What is the positive difference in the number of possible plates offered by the two options?

___2___ 3.) How many of the 7th roots of -1 lie in the 1st Quadrant of the complex plane?

__$-\frac{1}{2}$__ 4.) What is the exact value of $\sin\left(\frac{1819\pi}{6}\right)$?

__500π__ 5.) A cube of side length 10 is inscribed in a cylinder. What is the volume of the cylinder?

___20___ 6.) Omar gives $\frac{1}{4}$ of his gumballs to Manuel, $\frac{1}{5}$ of the remaining gumballs to Rajesh, then $\frac{1}{6}$ of the remaining gumballs to Dietrich. If he never had to split a gumball into parts during any of this process, what is the smallest number of gumballs with which he could have started?

__$(\mathbf{2},\mathbf{7})$__ 7.) Write the solution to the system as an ordered pair:

$7x + 3y = 35$ and $4x - 5y = -27$.

___48___ 8.) When $x^4 - 5x^3 + kx^2 + 7x - 5$ is divided by $x + 1$, the remainder is 42. What is the value of k?

___8___ 9.) Find the positive value of x so that the determinant of the given matrix has a value of 30. $\begin{bmatrix} 2 & 3 & x \\ 4 & x & 2 \\ 1 & 2 & x \end{bmatrix}$

___41%___ 10.) 50% of the 24 students in Mrs. A's class and 40% of the 25 students in Mrs. B's class are boys. In Mrs. C's class, the 6 boys represent 30% of the students. To the nearest integer, what percent of all students in the 3 classes are boys?

___3___ 11.) How many solutions does the equation $sin(x) = \frac{1}{6}x$ have?

__2,730__ 12.) Fifteen distinguishable runners run a race. How many ways are there to award gold, silver, and bronze medals (one of each)?

___2___ 13.) The set of possible values of *k*, for which the lines $y = \frac{3}{4}x - 3$ and $y = kx - 5$ intersect in the first Quadrant, can be expressed as the open interval (a, b). What is the value of $a + b$?

__$\frac{\mathbf{3}}{\mathbf{8}}$__ 14.) When 4 fair coins are flipped, what is the probability that 2 are heads up and 2 are heads down?

___89___ 15.) John made 85, 76, 78, and 72 on his first 4 tests. What score does he need to make on his fifth test in order to have an average of exactly 80?

__$\frac{\mathbf{19}}{\mathbf{2}}$__ 16.) What is the sum of ALL of the solutions to the equation:

$(2x^2 - 5x + 6)(x^2 - 7x + 4) = 0$

__2π__ 17.) Find the period of the function $f(x) = 3\cos\left(5x - \frac{\pi}{2}\right) + 4tan(2x - \pi)$.

999,471 18.) Calculate $(977)(1023)$.

__171__ 19.) What is the remainder when 11^7 is divided by 1000?

___6___ 20.) How many non-congruent rectangular prisms have positive integer side lengths and a volume of 24?

__$6\sqrt{13}$__ 21.) What is the distance between the points $(-12,4)$ and $(6,-8)$ in simplest radical form?

__-3___ 22.) Find: $\lim_{x \to 2} \frac{x^3 - 4x^2 + x + 6}{x^2 - 3x + 2}$.

__$\frac{\mathbf{16}}{\mathbf{3}}$__ 23.) Find the x-coordinate of the x-intercept of the line $y = \frac{-3}{2}x + 8$.

__64π__ 24.) Find the area enclosed by the ellipse given by the equation: $\frac{(x-4)^2}{64} + \frac{(y+3)^2}{16} = 2$.

__135___ 25.) A particular regular polygon is such that each interior angle is 8 times the degree measure of each exterior angle. How many diagonals does this regular polygon have?

1.) The only number that fits the description is the number "four" which has 4 letters! After the first 7 or eight numbers, it becomes pretty obvious that the number of letters is growing at a much slower rate than the number itself. The answer is **1**.

2.) Option 1 provides $26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ or $260 \cdot 10^4$ possible plates. Option 2 provides $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ or $676 \cdot 10^4$ possible plates. The difference is $416 \cdot 10^4$ or $\mathbf{4,160,000}$ plates.

3.) Clearly one of the roots is -1. The others are spread equally around the unit circle. Since $\frac{360}{7}$ is slightly over 51° (and this is good enough for an estimate), then we venture clockwise from 180° in increments, arriving at ROUGHLY 129°, 78°, 27°, -24°, etc. Note that **two** have the solutions are in Quadrant I, **namely** $\boldsymbol{cis\left(\frac{3\pi}{7}\right)}$ **and** $\boldsymbol{cis\left(\frac{\pi}{7}\right)}$**.**

4.) Since 2π represents an entire trip around the unit circle, I want to know the remainder when $\frac{1819\pi}{6}$ is divided by 2π. Now $\left(\frac{1819\pi}{6}\right) = 303\frac{1}{6}\pi$ which gives remainder $1\frac{1}{6}\pi$ or $\frac{7\pi}{6}$ when divided by 2π. Then $\sin\left(\frac{1819\pi}{6}\right) = \sin\left(\frac{7\pi}{6}\right) = -\frac{\mathbf{1}}{\mathbf{2}}$.

5.) The diameter of the cylinder is the diagonal of the square face atop the cube, so the diameter is $10\sqrt{2}$ and the radius is $5\sqrt{2}$. The height of the cylinder is the same as the height of the cube, so 10. $V_{cyl} = \pi r^2 h = \pi\left(5\sqrt{2}\right)^2(10) = \mathbf{500\pi}$.

6.) The smallest number with which he could begin the last phase is 6 gumballs. Since these 6 would represent $\frac{4}{5}$ of the gumballs prior to phase two, we multiply 6 by $\frac{5}{4}$ to work backwards. However, this does not give us an integer number of gumballs, but if the 6 were doubled to 12, then $12 \cdot \frac{5}{4} = 15$. The 15 represents $\frac{3}{4}$ of the original amount, which works to 20. Any positive multiple of 20 could have been the starting amount and kept him from splitting gumballs, which no one would ever want to do, whether because they are hard to cut, or because they don't like sharing, but **20** is the least initial number of gumballs.

7.) I did 5 times the first equation plus three times the second to arrive at $(\mathbf{2},\mathbf{7})$.

8.) When $x^4 - 5x^3 + kx^2 + 7x - 5$ is divided by $x + 1$, the remainder is 42. What is the value of k? Based on the Remainder Theorem, $f(-1) = 42$. Then $(-1)^4 - 5(-1)^3 + k(-1)^2 + 7(-1) - 5 = 1 + 5 + k - 7 - 5 = k - 6 = 42$ so $\boldsymbol{k} = \mathbf{48}$.

9.) When calculating the determinant of the matrix, we get $(2x^2 + 6 + 8x) - (x^2 + 8 + 12x) = x^2 - 4x - 2 = 30$. Subtract 30 from both sides to get $x^2 - 4x - 32 = 0$. Factor to $(x - 8)(x + 4) = 0$. This leads to the positive answer of $\boldsymbol{x} = \mathbf{8}$.

10.) 50% of the 24 students in Mrs. A's class and 40% of the 25 students in Mrs. B's class are boys. In Mrs. C's class, the 6 boys represent 30% of the students. To the nearest integer, what percent of all students in the 3 classes are boys? In Mrs. A's, 12 of 24 are boys. In Mrs. B's, 10 of 24 are boys. In Mrs. C's, 6 of 20 are boys. This gives 28 of 69 students are boys. Since 69 goes into 280 4 times with 4 left over, you can quickly arrive at **41%**.

11.) See how many intersection points exist for the graphs of $y = sin(x)$ and $y = \frac{1}{6}x$. The origin is an obvious intersection point. We also know the sine function has a maximum value of 1, and since the point $(6,1)$ is on the graph of $y = \frac{1}{6}x$, then we can spend our time on the sine graph. Sine increases quickly to the point $\left(\frac{\pi}{2}, 1\right)$, then returns to the x-axis at the point $(\pi, 0)$, crossing the graph of $y = \frac{1}{6}x$ at exactly one point in the process. Sine then returns to a max value at $\left(\frac{5\pi}{2}, 1\right)$, and since $\frac{5\pi}{2} > 6$, there is only one point with a positive x-coordinate that lies on both graphs. Lastly, both functions are odd, so there is a single corresponding point with a negative x-coordinate. These two, along with the origin, give 3 intersection points, and thus **3 solutions**.

12.) Fifteen (distinguishable) runners run a race. How many ways are there to award gold, silver, and bronze medals (one of each)? $15P3 = \frac{15!}{(15-3)!} = 15 \cdot 14 \cdot 13 = 210 \cdot 13 = \mathbf{2730}$.

13.) Graphing the lines, you will notice that if $k = \frac{5}{4}$ then the lines intersect on the x-axis, so we need $k < \frac{5}{4}$. If $k = \frac{3}{4}$, then the lines are parallel and won't intersect at all, so we need $k > \frac{3}{4}$. This gives the open interval $\left(\frac{3}{4}, \frac{5}{4}\right)$ and $\frac{3}{4} + \frac{5}{4} = \mathbf{2}$.

14.) When 4 fair coins are flipped, what is the probability that 2 are heads up, and 2 are heads down? Using the binomial probability model, $P(2\ heads) = 4C2 \cdot [P(heads)]^2 \cdot [P(tails)]^2 = \frac{4!}{2!2!} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2 = 6 \cdot \frac{1}{4} \cdot \frac{1}{4} = \mathbf{\frac{3}{8}}$.

15.) I want an average of 80, so I consider how each number differs from 80, so I change 85, 76, 78, and 72 to 5, -4, -2, and -8, which have a sum of -9. John must gain these points back on his last test. He needs to make an **89**.

16.) What is the sum of ALL of the solutions to the equation: $(2x^2 - 5x + 6)(x^2 - 7x + 4) = 0$ $2x^2 - 5x + 6 = 0$ has two imaginary solutions with a sum of $\frac{5}{2}$. $x^2 - 7x + 4 = 0$ has two real solutions with a sum of 7. Since all of the roots are then distinct, we do $\frac{5}{2} + 7$ to get $\mathbf{\frac{19}{2}}$.

17.) Find the period of the function $(x) = 3\cos\left(5x - \frac{\pi}{2}\right) + 4tan(2x - \pi)$. The period of the first component is $\frac{2\pi}{5}$, and the period of the second component is $\frac{\pi}{2}$. The LCM of these two periods is $\mathbf{2\pi}$.

18.) Calculate $(977)(1023)$. Consider $(1000 - 23)(1000 + 23) = 1000^2 - 23^2 = 1{,}000{,}000 - 529 = \mathbf{999{,}471}$.

19.) Think about the expansion of $(10 + 1)^7$. The first 5 terms will involve 10^7 through 10^3, so they just provide multiples of 1000, or remainder zero. The last 3 terms are $\frac{7!}{2!5!}(10)^2(1)^5 + \frac{7!}{1!6!}(10)^1(1)^6 + \frac{7!}{0!7!}(10)^0(1)^7$ or $2100 + 70 + 1$ or 2171, so the remainder when divided by 1000 is $\mathbf{171}$.

20.) Translated a little, the question becomes, "how many distinguishable groups of 3 integers have a product of 24?" So as not to duplicate, I list ordered triples where $x \le y \le z$. I begin with (1,1,24), then (1,2,12), (1,3,8), (1,4,6), then (2,2,6) and (2,3,4). There are only these **6 sets of dimensions** which satisfy the conditions. (You could also do the problem by picking a value for x, then considering the number of pairs of positive integer factors of $\frac{24}{x}$ that still satisfy $x \leq y \leq z$, but this way isn't time effective for such a small number of options.

21.) The vector $\langle 18, -12\rangle$ takes us from one point to the other, and the magnitude of the vector is $\sqrt{(18)^2 + (-12)^2} = \mathbf{6\sqrt{13}}$ or just note that the magnitude of $\langle 18, -12\rangle$ is 6 times the magnitude of $\langle 3, -2\rangle$. Or just use the distance formula, but that isn't as fun!

22.) Initially, you will get $\frac{0}{0}$, so we can simplify by reducing synthetically by $(x-2)$ to get $\lim_{x \to 2} \frac{x^2-2x-3}{x-1}$. The limit of this expression is **-3**. (You can also use l'Hospital's rule to get the diminished expression and the same result, but I don't want to force anyone to use Calculus.)

23.) Let $y = 0$. This gives the equation $0 = \frac{-3}{2}x + 8$, which has the solution $\boldsymbol{x} = \mathbf{\frac{16}{3}}$.

24.) Find the area of the ellipse given by the equation: $\frac{(x-4)^2}{64} + \frac{(y+3)^2}{16} = 2$. First divide by 2, to get the equation $\frac{(x-4)^2}{128} + \frac{(y+3)^2}{32} = 1$. This means $a^2 = 128$ so $a = 8\sqrt{2}$ and $b^2 = 32$ so $a = 4\sqrt{2}$. The area of the ellipse is $A = ab\pi = (8\sqrt{2})(4\sqrt{2})\pi = \boldsymbol{64\pi}$.

25.) Let the exterior angle be x and the interior be $8x$. We know then that $x + 8x = 180$, so $x = 20$, which gives an exterior angle of 20°. Since all of the exterior angles (one at each vertex) have a sum of 360°, then there must be 18 vertices. The number of diagonals in an *n*-gon is given by $\frac{n(n-3)}{2}$ and in this case is $\frac{18(18-3)}{2} = \frac{18(15)}{2} = \mathbf{135}$.